Modeling vector-borne diseases

Ashley Hill

Vector-borne diseases

- Mosquitos
  - Malaria
  - Heartworm
  - West Nile Virus
  - Yellow fever
- Ticks
  - Lyme disease
  - Ehrlichiosis
  - Tick-borne relapsing fever

Standard model based on malaria

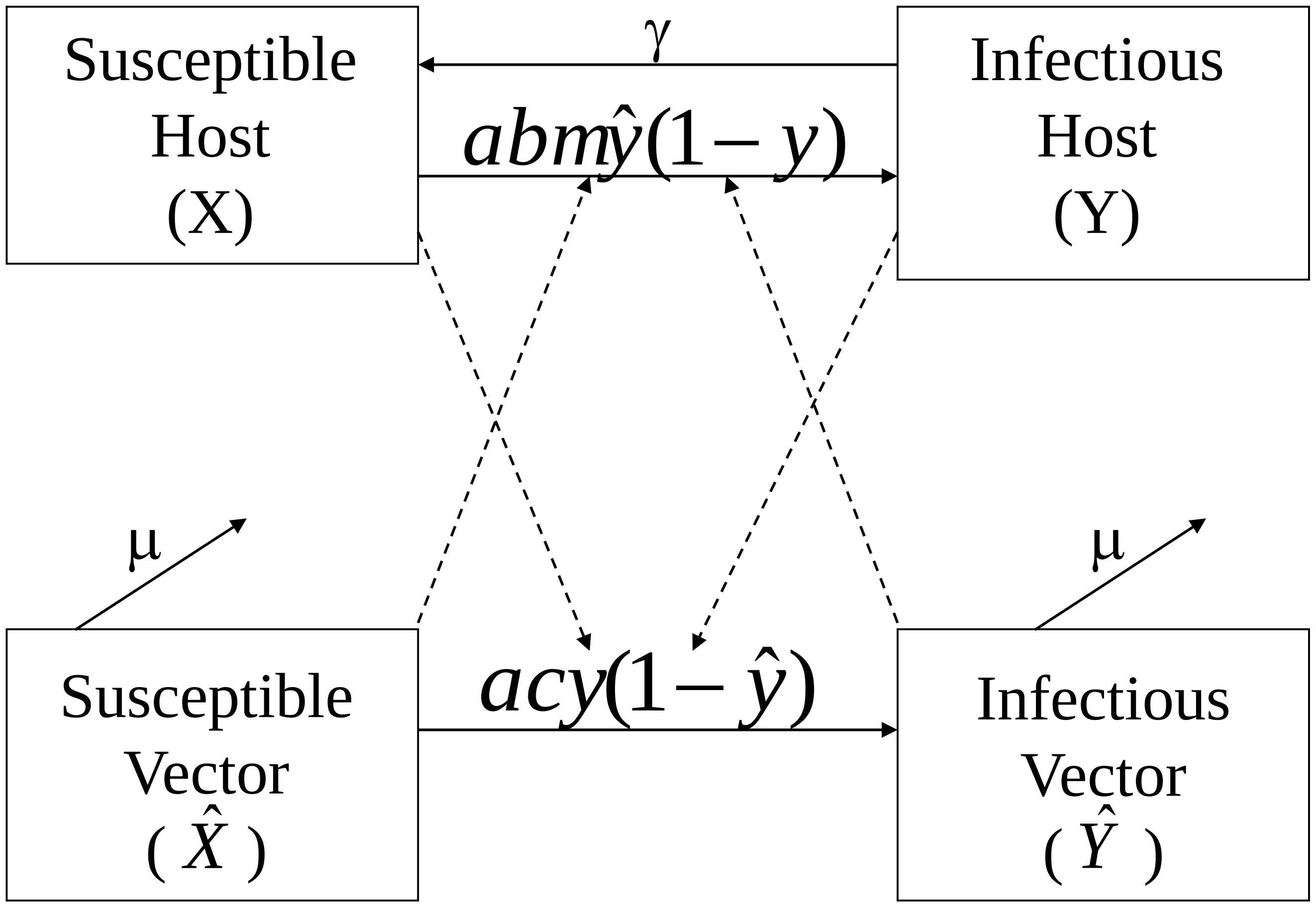
- Epidemiology of malaria
  - Several disease agents
    - Protozoan parasites
      - Plasmodium vivax, P. falciparum, P. ovale, P. malariae
  - Transmitted by female mosquitoes via blood meal
  - Mosquitoes infectious after disease agent completes growth cycle
    - 9-21 days at 25 C (shorter at higher temps)
  - 7-30 day incubation period in host after bite before symptoms appear
  - Parasites can remain dormant in host and re-activate periodically

Assumptions of basic model

- No host immunity
- No disease-associated mortality in host or vector
- No latent state for host or vector
- Constant population size
  - Birth rate = death rate
- Newborns are susceptible
- Vector life-span <<< host life-span

- Concept: 2 separate populations (host & vector) that affect each other

Basic vector-borne disease model



Definitions

- a = # bites on host per time period
- b = vector infectivity
  - P(host infection | vector infectious)
- m = biting vector:host ratio
- $\hat{y}$ = proportion of infectious vectors (vector disease prevalence)
- y = proportion of infectious hosts (host disease prevalence)

Definitions, continued

- c = host infectivity
  - P(vector infection | host infectious)
  - proportion of susceptible vector bites on infectious host that produce vector infection
- γ = rate of recovery from infection
  - = 1/D (duration of infection)
- μ = rate of death in vector population
  - = 1/L (vector life expectancy)
- N = Density of host population
  - N = X + Y
- $\hat{N}$ = Density of vector population
  - $\hat{N} = \hat{X} + \hat{Y}$

What affects host incidence?

- Host incidence increases with
  - Bites/host in time period
  - Vector infectivity
  - Vector:host ratio
  - Disease prevalence in vector
- Host incidence decreases with
  - Disease prevalence in host (decreased probability that bitten host is uninfected)
- Host prevalence decreases with
  - Duration of infection

What affects vector incidence?

- Vector incidence increases with
  - Bites/host in time period
  - Host infectivity
  - Disease prevalence in host
- Vector incidence decreases with
  - Disease prevalence in vector (decreased probability that biting vector is uninfected)

Which parameters might be targets for control measures?

- Bites/host (a)
  - Bug sprays, i.e. DEET products
- Vector infectivity (b)
  - Host vaccine programs
- Vector:host ratio (m)
  - Environmental insect control programs

Rate of change in host prevalence:

$$abm\hat{y}(1-y)-\gamma y$$

Incidence rate
– recovery rate

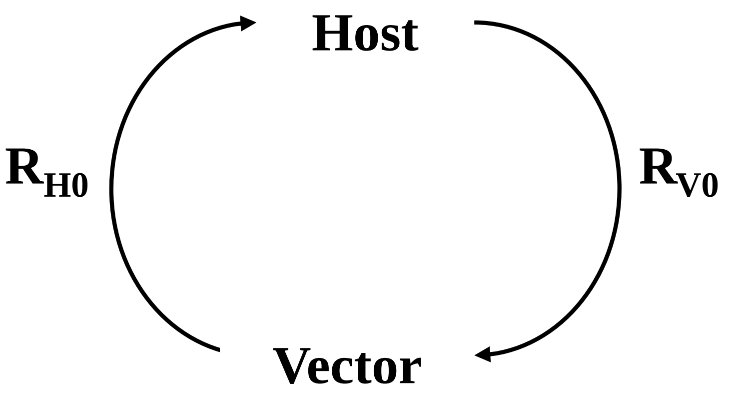
Rate of change in vector prevalence:

$$acy(1-\hat{y})-\mu\hat{y}$$

Incidence rate
– death rate

R_0 for vector-borne diseases

- Number of secondary cases per primary case in a susceptible population
- Product of host-to-vector transmission (R_{V0}) and vector-to-host transmission (R_{H0})



R_0 for vector-borne diseases

- Number of vector cases per host (R_{V0})
  - amc/γ
    - Bites/vector * vectors/host * host infectivity * duration of host infectivity
- Number of host cases per vector (R_{H0})
  - ab/μ
    - Bites/vector * vector infectivity * lifespan of vector

R_0 for vector-borne diseases

- Overall rate is R_{V0} * R_{H0}:

$$\frac{amc}{\gamma} * \frac{ab}{\mu} = \frac{ma^2bc}{\mu\gamma} = R_0$$

- Note the importance of “a”:
  - Number of bites on host

Stability (endemic state)

- Occurs when rate of change in host and vector populations is 0
- Useful for determining
  - Whether an endemic state can develop
  - Vector / host prevalence in endemic state

Stability

- Host stability: $$abm\hat{y}(1-y)-\gamma y=0$$
$$abm\hat{y}=\frac{\gamma y}{1-y}$$
- Vector stability: $$acy(1-\hat{y})-\mu\hat{y}=0$$
$$acy=\frac{\mu\hat{y}}{1-\hat{y}}$$
- Solve for y and ŷ to determine equilibrium prevalence

Prevalence at equilibrium

- Host: $$y^* = \frac{(R_0 - 1)}{\left[R_0 + \frac{ac}{\mu}\right]}$$

- Vector: $$\hat{y}^* = \left(\frac{(R_0 - 1)}{R_0}\right)\left(\frac{\frac{ac}{\mu}}{1 + \frac{ac}{\mu}}\right)$$

Macdonald's stability index

- ac/μ
- Larger value suggests endemic disease
- Smaller value suggests epidemic outbreaks that die off
- Larger value when:
  - Many bites/host
  - High P(vector infection | host infected)
  - Long vector lifespan
- Reported values from 0.47 to 4.9 for malaria in Africa

Spreadsheet model – user inputs

User inputs		
Time step (in 10-day units)	dt	0.1
Vector biting rate per day	a	4
Proportion of bites that infect host	b	0.1
Proportion of bites that infect vector	c	0.1
Life expectancy of vectors	L	2
Duration of host infectiousness (% vector life expectancy)	D	3
Number of hosts	N	1000
Number of vectors	Nhat	100000
Infected host population t=0	Y_0	100
Infected vector population t=0	$Yhat_0$	1000

Spreadsheet model – calculated parameters

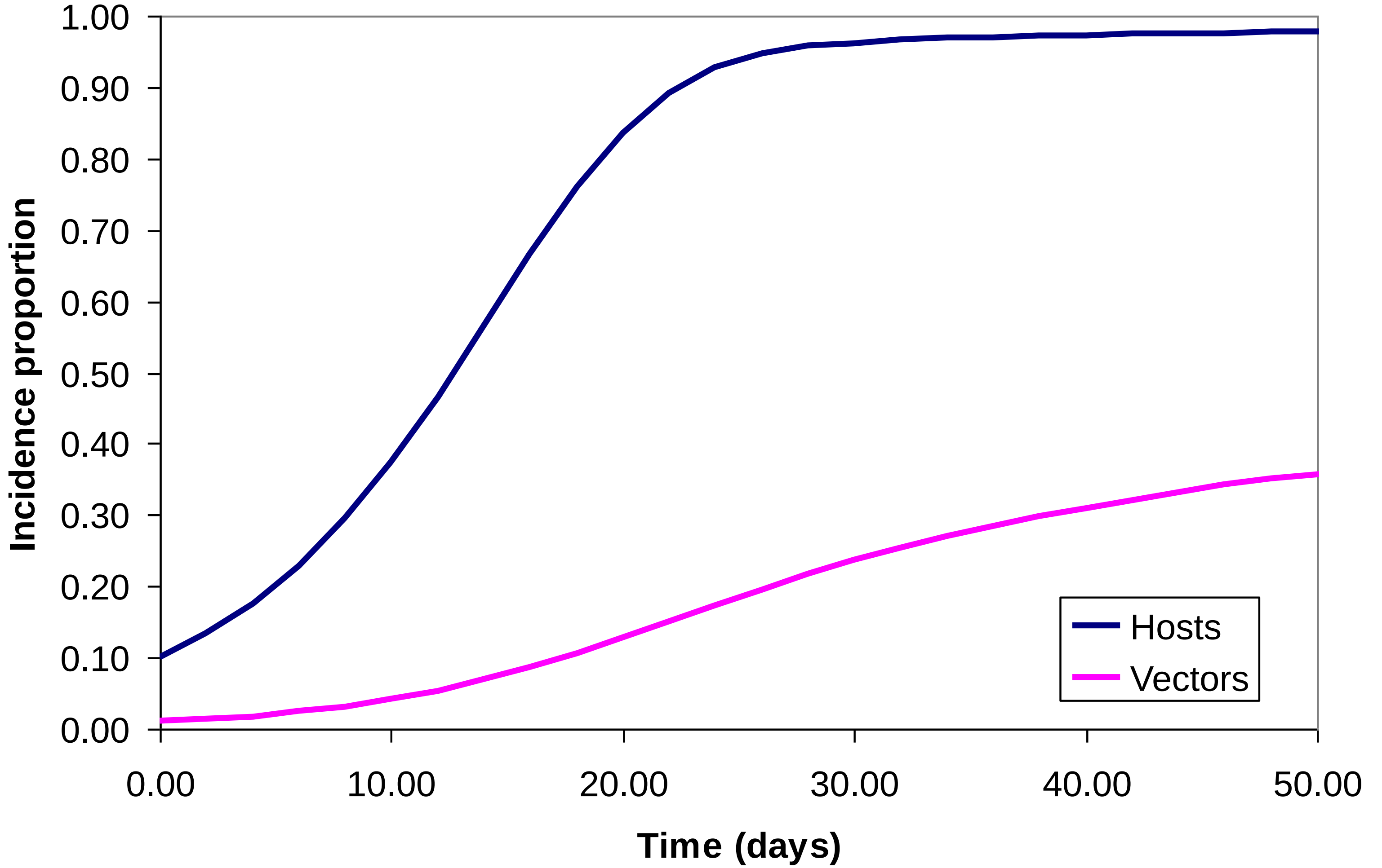
Parameters calculated from user inputs		
Vector-to-host density	m	100
Vector mortality rate	mu	0.5
Host infectious recovery rate	gamma	0.33333
Host basic reproductive rate	R_{H0}	0.8
Vector basic reproductive rate	R_{V0}	120.0
Basic reproductive rate	R_o	96.0
MacDonald's stability index		0.8

Spreadsheet model - exploration

Spreadsheet model - results

Equilibrium values		
Parameter	**CALCULATED**	**OBSERVED**
Number infected hosts at equilibrium	981.4	981.4
Number infected vectors at equilibrium	43981.5	43981.5
% hosts infected at equilibrium	0.981	0.981
% vectors infected at equilibrium	0.440	0.440

Spreadsheet model - results



Expansion beyond the basic model

- Latent periods in vector and host
- Seasonal changes in mosquito population
- Co-infections in humans
- Acquired immunity

References

- Anderson RM, May RM, Infectious Diseases of Humans: Dynamics and Control. 1991. Oxford University Press. Pp 392-419.